**Cubic map**

1. If we study the equation xn+1  = rx2n (1-xn), and we make the derivative ( r\*(2x -3x2)) we obtain two critical points, x=0 and x=2/3, being the second one the maximum of this equation when r >0, knowing that we calculate the maximum value of r= 6,75.
2. The fixed obtained are, searching for the root of x\*:

* X\* = 0
* X\* =
* X\*=

1. If we see of those last points, we can observe that r must be greater than 4, if not the square root will be negative. Then introducing the values of X\* in the derivative of the equation, we obtain f’(x\*) = , the absolute value of this equation must be less than one in order to be stable. In last two cases r must be r >4 as we noticed before, in first case the solution is always 0, so it always stable.

In this case X\* = , we obtain other point r<5.33.

Interfaz de usuario gráfica, Aplicación

Descripción generada automáticamente

1. Here we can see some examples of the model, being blue r=3, red r=4, yellow r=6 green r=7

Interfaz de usuario gráfica, Aplicación

Descripción generada automáticamente

1. Cobweb plotting

r = 3

Interfaz de usuario gráfica

Descripción generada automáticamente

Population is decreasing

r=4

Interfaz de usuario gráfica, Aplicación, Word

Descripción generada automáticamente

With r=4 we can see that the population is stable along the generations.

r=5,33

Interfaz de usuario gráfica

Descripción generada automáticamente

In the first graph we can see that from x=0.5 we obtain an increase of the population, reaching a cyclic point, but with values of x much lower we will obtain a different result as we can appreciate in the next graph, where we can see how the population decrease.

r=6

Interfaz de usuario gráfica, Gráfico

Descripción generada automáticamente Gráfico

Descripción generada automáticamente

With r=6 we can see a loop, we have an oscillating population. In my graph the behavior it is not really appreciated so I adjunct another graph made with geogebra.

r=7

Gráfico

Descripción generada automáticamente

Here we can see that we don’t obtain any stable point, the system collapses (it jumps into negative values of x and from there it diverges towards minus infinity).

Lyapunov exponent

Interfaz de usuario gráfica

Descripción generada automáticamente

Interfaz de usuario gráfica, Aplicación

Descripción generada automáticamente

I made this graph changing the code in this website <http://systems-sciences.uni-graz.at/etextbook/sw2/lyapunov.html>, in the first graph we can see the Cubic’s map Lyapunov exponent and in the second one we have the comparation between the bifurcation diagram and Lyapunov exponent, so we can appreciate how fast the cubic map system reach for chaos, and how the exponent is greater as we approach the branching points studied before.